Recapitulation

So far we have read logical connectives, truth table. For 5 basic connectives AND ($\wedge$), OR ($\vee$), NOT ($\sim$), Implication ($\rightarrow$) and bicondition ($\leftrightarrow$), we found Truth Tables (TT) too.

We learnt that AND is also called conjuction, OR is also called Disjunction. Symbols of AND, OR, NOT are not unique. Like

AND .... $\wedge$, $\bullet$ [Ex: $P_1 \wedge P_2$ is same as $P_1 \bullet P_2$]

OR .... $\vee$, $+$ [Ex: $P_1 \vee P_2$ is same as $P_1 + P_2$]

NOT .... $\sim$, $-$, $'$ [Ex: $\sim P_1$, $-P_1$; P_1' are all same]

Implication .... $\rightarrow$, $\Rightarrow$ [Ex: $P_1 \rightarrow P_2$, $P_1 \Rightarrow P_2$ are same]
(if P_1 then P_2)

Bicondition .... $\leftrightarrow$, $\Leftrightarrow$, [Ex: $P_1 \leftrightarrow P_2$, $P_1 \Leftrightarrow P_2$ are same]
(P_1 iff P_2)

We have learnt that any proposition ~~or statement~~ has two values: T or F OR 1 or 0.
We have learnt how to create ToT of a statement.

Now we learn how to check whether two statements are logically equivalent —

Two ~~logical~~ statements are logically equivalent if they have same truth value for same situation.

Ex

Statement 1 – It will not rain or snow

Statement 2 – It will not rain and it will not snow

Sol^n

Let P_1 : It will ~~not~~ rain

P_2 : It will snow

statement 1 : $\sim (P_1 \vee P_2)$

statement 2 : $\sim P_1 \wedge \sim P_2$

Let us find T.T of two statements.

Statement 1 :

P_1	P_2	$P_1 \vee P_2$	$\sim (P_1 \vee P_2)$
1	1	1	0
1	0	1	0
0	1	1	0
0	0	0	1

statement 2 :

P_1	P_2	$\sim P_1$	$\sim P_2$	$\sim P_1 \wedge \sim P_2$
1	1	0	0	0
1	0	0	1	0
0	1	1	0	0
0	0	1	1	1

From above two T.T, we see that for same combination of P_1 & P_2, final truth values are same.

Hence statement 1 & statement 2 are logically equivalent.

Here we must know that Boolean Algebra is associated with propositional logic and so, De' Morgan's Law is applicable here.

$$\boxed{\begin{aligned} \sim(P_1 \vee P_2) &\equiv \sim P_1 \wedge \sim P_2 \\ \sim(P_1 \wedge P_2) &\equiv \sim P_1 \vee \sim P_2 \end{aligned}}$$

Ex We have already proved that (1) $P_1 \rightarrow P_2$ is equivalent to $(\sim P_1 \vee P_2)$, (2) $\sim\sim P_1$ is equivalent to P_1, (3) $\sim(P \rightarrow Q)$ is equivalent to $P \wedge (\sim Q)$.

Practice at home.

Now we check two more connectives, NAND and NOR. NAND is equivalent to Not AND i.e., $\sim(P_1 \wedge P_2)$ and NOR is equivalent to Not OR i.e., $\sim(P_1 \vee P_2)$.

Lets explore NAND $(\sim(P_1 \wedge P_2))$. T.T is

P_1	P_2	$P_1 \wedge P_2$	$\sim(P_1 \wedge P_2)$

T. T of NAND

P_1	P_2	$P_1 \wedge P_2$	$\sim(P_1 \wedge P_2)$
1	1	1	0
1	0	0	1
0	1	0	1
0	0	0	1

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T. T of NOR

P_1	P_2	$P_1 \vee P_2$	$\sim(P_1 \vee P_2)$
1	1	1	0
1	0	1	0
0	1	1	0
0	0	0	1

Complete Set of Connectives :

A set of connectives is called a complete set of connectives if any formula can be expressed by using that set of connectives only.

For example, $(\sim, \vee)$ is complete set of connectives.

Because $\wedge$ can be expressed in terms of $(\sim, \vee)$ by De' Morgan's Law.

Ex

$$(P_1 \wedge P_2) \equiv \sim P_1 \vee \sim P_2$$

Similarly, $(\sim, \wedge)$ is complete set of connectives.

But $(\wedge, \vee)$ is not complete because $\sim$ can't be expressed by using $(\wedge, \vee)$.

Here we know an interesting fact :- NAND is also called Sheffer stroke and its symbol is ~~written~~ is $|$.

We can express other connectives in terms of Sheffer Stroke $(|)$.

$\sim A$ T.T is

A	$\sim A$
1	0
0	1

—— ①

From ① & ②, See that

$\sim A \equiv A \mid A$

$A \mid A$ --- T.T is

$\equiv (A \text{ NAND } A)$

$\equiv \sim (A \wedge A)$

A	A	$A \wedge A$	$\sim(A \wedge A)$
1	1	1	0
0	0	0	1

—— ②

$$A \vee B \equiv \sim(\sim A \wedge \sim B) \equiv \sim((A|A) \wedge (B|B))$$

$$\cancel{\equiv \sim\sim(A|A) \vee \sim\sim(B|B)}$$

$$\equiv (A|A) \,|\, (B|B)$$

Similarly try to show that $\wedge$ can be expressed by $|$ only.

So we can say that NAND i.e., Sheffer's Stroke i.e., $|$ is a complete set of connectives.

Home Exercise-1

Home Exercise – 1

1) Prove that statements $\sim(P \to Q)$ and $P \wedge \sim Q$ are logically equivalent a) by using Truth Table and b) without using Truth Tables.

2) Check whether $(P \vee Q) \to R$ and $(P \to R) \vee (Q \to R)$ are logically equivalent.

3) Try to express AND ($\wedge$) in terms of Sheffer Stroke i.e., $|$.

4) Try to express NOR in terms of Sheffer stroke i.e., $|$.