[... for this topic]

Exogenous (Old) Growth Theory : Phase I

The Harrod Model

~~Thus~~ Harrod's model is a Keynesian model of economic growth. Harrod has developed an axiomatic basis of the theory and to give it a concrete shape he has suggested three propositions.

1) The level of community's income is the most important determinant of its supply of saving
2) The rate of increase of its income is an important determinant of its demand for saving and
3) Demand is equal to supply.

To develop his model Harrod has ~~developed~~ considered a marriage of the 'acceleration principle' and the multiplier theory.

Following Harrod's assumptions about lags we thus specify

$$S_t = s\,Y_{t-1} \quad \text{——— (1)}$$

(1) ⇒ Saving function and s is the marginal propensity to save (S_t ⇒ saving at time t)
Y_t ⇒ Income at time t.

The investment function is specified as

$$I_t = v\,(Y_t - Y_{t-1}) \quad \text{——— (2)}$$

where I_t ⇒ Investment at time
v ⇒ accelerator
Y_t ⇒ Income (or output) at time t.

As $S_t = I_t$ at equilibrium

$$\Rightarrow Y_t - C_t = I_t$$

or, $$Y_t = C_t + I_t \quad \text{——— (3)}$$

We now focus on the supply side

Harrod defined the concept of warranted rate of growth. The warranted rate of growth is taken to be that rate of growth which, if it occurs, will leave all the parties satisfied that they have produced neither more nor less than the right amount. It is ~~less~~ basically 'equilibrium' rate of growth but Harrod has preferred to use the term 'warranted' instead of 'equilibrium'

~~Thus, warranted rate of growth rate (g_w) implies~~

Harrod has specified the determinants of warranted rate of growth and has referred to warranted rate of growth (g_w) in terms of the following fundamental equation

$$g_w = \frac{s}{v} \qquad \text{——— (4)}$$

where s is the marginal propensity to save and v implies the state of technology or the accelerator or the incremental capital-output ratio. [It is to be noted that $I_t = K_t - K_{t-1}$ where K_t is capital stock at time t.

$$I_t = v(Y_t - Y_{t-1})$$

$$\text{or, } K_t - K_{t-1} = v(Y_t - Y_{t-1})$$

$$\text{or, } v = \frac{K_t - K_{t-1}}{Y_t - Y_{t-1}} \;]$$

The actual rate of growth (g) is defined by

$$g = \frac{Y_t - Y_{t-1}}{Y_{t-1}} \qquad \text{——— (5)}$$

Equation (3) $\Rightarrow$

$$S_t = I_t$$

$$\Rightarrow \; sY_{t-1} = v(Y_t - Y_{t-1})$$

$$\Rightarrow \; \frac{s}{v} = \frac{Y_t - Y_{t-1}}{Y_{t-1}}$$

Thus $$g = \frac{Y_t - Y_{t-1}}{Y_{t-1}} = \frac{s}{v} = g_w \qquad \text{——— (6)}$$

Equation (6) implies short run equilibrium condition. Thus, the warranted rate of growth is self perpetuating. Once it is achieved this rate of growth will be maintained.

We have mentioned that once the warranted rate of growth is achieved it will continue. According to Harrod this is due to interaction of multiplier and accelerator.

Suppose we consider a temporary divergence of the actual rate of growth from the warranted rate of growth. According to Harrod such a temporary divergence from the warranted growth path would not be self-correcting. In fact, once the actual growth rate falls below the warranted growth rate, there will be forces in the economy which will push it further away from the warranted growth rate. The same type of repercussions, according to Harrod, will occur if the actual rate of growth exceeds the warranted rate of growth. This is known as 'knife-edge instability' in Harrod's model.

We first consider

$$u_t = I_t - S_t \qquad (7)$$

where u_t is a random disturbance term so that when $I_t > S_t$ we have $u_t > 0$ and when $I_t < S_t$ we have $u_t < 0$.

Actually $u_t > 0 \Rightarrow I_t > S_t \Rightarrow$ excess of aggregate demand over aggregate supply

Again $u_t < 0 \Rightarrow I_t < S_t \Rightarrow$ excess of aggregate supply over aggregate demand.

We also assume

$$g_{t+1} = g_t + f(u_t) \quad \text{———} (8)$$

where $f(u_t) \gtreqless 0$ according as $u_t \gtreqless 0$.

We start from a positive temporary disturbance like $u_t > 0$

$$\Rightarrow \quad I_t > S_t$$

$$\Rightarrow \quad v(Y_t - Y_{t-1}) > s\, Y_{t-1}$$

$$\Rightarrow \quad \frac{(Y_t - Y_{t-1})}{Y_{t-1}} > \frac{s}{v}$$

$$\Rightarrow \quad g_t > g_w \quad \text{———} (9)$$

when $u_t > 0 \Rightarrow f(u_t) > 0$

$$\Rightarrow \quad g_{t+1} > g_t \quad (\text{from } (8))$$

$$\Rightarrow \quad g_{t+1} > g_t > g_w \quad \text{———} (10)$$

Rewriting (8) as

$$g_{t+2} = g_{t+1} + f(u_{t+1}) \quad \text{———} (8')$$

with $f(u_{t+1}) \gtreqless 0$ according as $u_{t+1} \gtreqless 0$

~~$\Rightarrow$~~ Thus,

$$I_{t+1} > S_{t+1}$$

$$\Rightarrow v(Y_{t+1} - Y_t) > s\, Y_t$$

$$\Rightarrow \quad g_{t+1} > \frac{s}{v}$$

when $u_{t+1} > 0$, $(8') \Rightarrow g_{t+2} > g_{t+1} \Rightarrow g_{t+2} > g_{t+1} > g_t > g_w$ ——— (11)

Proceeding in this way we ultimately get

~~$g_{t+\theta} > g_{t+(\theta-1)} >$~~

$$g_{t+\theta} > g_{t+(\theta-1)} > \cdots\cdots > g_{t+2} > g_{t+1} > g_t > g_w = \frac{s}{v}$$

———— (12)

Similarly if we consider a negative disturbance like $u_t < 0$

we have $I_t < S_t$

and we ultimately end at

$$g_{t+\theta} < g_{t+(\theta-1)} < \cdots\cdots < g_{t+2} < g_{t+1} < g_t < g_w = \frac{s}{v}$$

———— (13)

We now focus on the longrun equilibrium condition in Harrod's model. The long run equilibrium condition is related to the natural rate of growth. The natural rate of growth is given by the rate of growth of population ~~to~~ 'n'. Thus

$$\frac{L^S_{t+1} - L^S_t}{L^S_t} = n = \frac{L^S_t - L^S_{t-1}}{L^S_{t-1}} \quad \text{— (14)}$$

$$\Rightarrow L^S_{t+1} = L^S_t(1+n) \quad \text{— (15)}$$

and $L^S_t = L^S_{t-1}(1+n)$ — (16)

Using (16) we can write (15) as

$$L^S_{t+1} = L^S_{t-1}(1+n)^2$$

Proceeding just like above we get

$$L^S_{t+1} = L^S_0(1+n)^{(t+1)} \quad \text{———— (17)}$$

and $L_t^S = L_0^S (1+n)^t$ ——— (18)

The demand for labour is given by

$$L_t^d = bY_{t-1} \quad \text{———} \quad (19)$$

The demand for labour in case of firm at time 't' is dependent on realised output of the previous period. Here 'b' is the constant labour-output ratio.

On the basis of equation (19) we can write

$$L_{t+1}^d = bY_t \quad \text{———} \quad (20)$$

The flow labour market equilibrium condition is given by

$$L_{t+1}^d - L_t^d = L_{t+1}^S - L_t^S \quad \text{———} \quad (21)$$

The stock labour market equilibrium condition is given by

$$L_t^d = L_t^S \quad \text{———} \quad (22)$$

Thus at equilibrium

$$L_t^S = L_t^d = bY_{t-1} \quad \text{———} \quad (23)$$

Using equations (17) and (18) we can write

$$L_{t+1}^S - L_t^S = L_0(1+n)^{t+1} - L_t^S$$

$$= L_0(1+n)^t(1+n) - L_t^S$$

$$= L_t^S(1+n) - L_t^S \quad \text{(Using (18))}$$

$$= nL_t^S = nbY_{t-1} \quad \text{(Using (23))}$$

——— (24)

Using (19), (20) and (24) in equation (21) we get

$$bY_t - bY_{t-1} = nbY_{t-1}$$