or, $$\frac{Y_t - Y_{t-1}}{Y_{t-1}} = n \qquad \text{——} (25)$$

we know that from short run equilibrium

that $$g_t = \frac{Y_t - Y_{t-1}}{Y_{t-1}} = g_w = \frac{s}{v}$$

Hence (25) implies long run equilibrium condition as

$$g_t = g_w = \frac{s}{v} = g_n = n \qquad \text{——} (26)$$

where $g_t \Rightarrow$ actual rate of growth
$g_w \Rightarrow$ warranted rate of growth
$g_n \Rightarrow$ natural rate of growth

Thus natural rate of growth is that rate of growth which is required to maintain full employment in the economy which we achieve when equation (26) is satisfied

[Note: When equation (26) is satisfied we have demand for labour = supply of labour]

The question that arises is what happens when g_w is different from g_n? The result is that when $g_w > g_n$ we have unemployment of capital though there is full employment of labour. When $g_w < g_n$ we have full employment of capital but unemployment of labour. To capture such disequilibrium situation we consider

$$g_t = \min(g_w, g_n) \qquad \text{——} (27)$$

When

$$g_w > g_n$$

we have $g_n = g_t$

Thus, $\frac{s}{v} > g_n$

$$\Rightarrow \frac{s}{v} > g_t$$

$$\text{or, } \frac{s}{v} > \frac{Y_t - Y_{t-1}}{Y_{t-1}}$$

$$\text{or, } sY_{t-1} > v(Y_t - Y_{t-1})$$

$$\text{or, } S_t > I_t \quad \text{——— (28)}$$

(28) $\Rightarrow$ supply of capital > Demand for capital

$\Rightarrow$ unemployment of capital

When $g_n > g_w$

we have $g_w = g_t$

Thus, $n > g_w$

$$\Rightarrow n > g_t$$

$$\Rightarrow n > \frac{Y_t - Y_{t-1}}{Y_{t-1}}$$

$$\Rightarrow nY_{t-1} > (Y_t - Y_{t-1})$$

$$\text{or, } nbY_{t-1} > b(Y_t - Y_{t-1})$$

$$\text{or, } L^s_{t+1} - L^s_t > L^d_{t+1} - L^d_t \quad \text{——— (29)}$$

Relation (29) $\Rightarrow$

Supply of labour > Demand for labour

$\Rightarrow$ unemployment of labour force.

When $g_w > g_n$ (as $g_n = g_t$) it is impossible to maintain the warranted rate of growth. As ~~of~~ a result of this the investors will be disappointed and will expect the growth rate to fall in future. Hence they will invest less which will result in excess supply of capital stock in the economy without proper utilization. This causes unemployment of capital stock.

When $g_w < g_n$ (as $g_w = g_t$) we find that the natural rate of growth is very high and it is not possible for the economy to maintain full employment. In this case unemployment is a chronic problem.

Thus, Harrod's model provides a pessimistic view of the economy where balanced growth is highly unlikely to take place.

The Domar Model

view of the economy where balanced growth
highly unlikely to take place.

The Domar Model

The basic premises of the Domar model are as follows:

Any change in the rate of Investment flow per year, I, will produce a dual effect: it will affect the aggregate demand as well as productive capacity of the economy.

The demand effect of a change in 'I' operates through the multiplier process and is assumed to work instantaneously. Thus an increase in I raises the rate of income flow per year, Y, by a multiple of the increment in I. The multiplier $k = \frac{1}{\alpha}$ is given by the constant marginal propensity to save, α.

We define σ as the potential social average investment productivity where

$$\sigma = \frac{dP/dt}{I} \qquad (1)$$

where P ⇒ productive capacity

(1) ⇒

$$\frac{dP}{dt} = I\sigma \qquad (2)$$

$\frac{dP}{dt}$ is a function of I and σ and not of $\frac{dI}{dt}$. So even if $\frac{dI}{dt} < 0$, we find $\frac{dP}{dt}$ is always positive as long as I and σ are positive. Equation (2) implies the increase in productivity is essentially the supply side of our system.

On the demand side we have the multiplier theory so that

$Y = \frac{1}{\alpha} . I$ where α is the marginal propensity to save. Thus

$$\frac{dY}{dt} = \frac{dI}{dt} \cdot \frac{1}{\alpha} \qquad (3)$$

Let the economy be in an equilibrium position at time point zero. Thus

$$P_0 = Y_0 \quad \text{——— (4)}$$

To retain the equilibrium position, we must have

$$\frac{dP}{dt} = \frac{dY}{dt} \quad \text{——— (5)}$$

Substituting equations (2) and (3) in equation (5) we obtain our fundamental equation

$$I\sigma = \frac{dI}{dt} \cdot \frac{1}{\alpha} \quad \text{——— (6)}$$

Solution of (6) gives us

$$I = I_0 e^{\alpha \sigma t} \quad \text{——— (7)}$$

$\alpha\sigma$ is the equilibrium rate of growth. So long as it remains constant, the maintenance of fullemployment requires investment to grow at a constant compound interest rate.

We assume

(i) $\frac{I}{Y} = \alpha \Rightarrow aps = mps = \alpha$

(ii) $\frac{P}{K} = s \Rightarrow$ ratio ~~pa~~ of productive capacity to capital.

We assume that investment grows at a constant percentage rate, r, which, however, is not necessarily equal to the ~~equilibrio~~ equilibrium rate $\alpha\sigma$.

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We now consider various cases

[Note $\sigma = \frac{dP/dt}{\sigma I} = \frac{dP/dt}{dK/dt}$

$s = \frac{P}{K}$

$\sigma = s \Rightarrow \frac{dP/dt}{dK/dt} = \frac{P}{K}$]

Case I : $\sigma = s$

Since $I = I_0 e^{rt}$ we have

$$K = K_0 + I_0 \int_0^t e^{rt'} dt'$$

$$= K_0 + \frac{I_0}{r}(e^{rt} - 1) \qquad (8)$$

If t becomes large, K will approach the expression $\frac{I_0}{r} e^{rt}$ —— (9)

so that capital will also grow at a rate approaching r. $\left[\frac{dK}{dt} = \frac{I_0}{r} \cdot r e^{rt} = I_0 e^{rt}\right]$

As $Y = (1/\alpha) I_0 e^{rt}$, the ratio of income to capital is

$$\frac{Y}{K} = \frac{(1/\alpha) I_0 e^{rt}}{K_0 + (I_0/r)(e^{rt} - 1)} \qquad (10)$$

and

$$\lim_{t \to \infty} \frac{Y}{K} = \frac{r}{\alpha} \qquad (11)$$

Thus so long as r and α remain constant (or changes in the same proportion) no 'deepening' of capital takes place.

Substituting $K = \frac{P}{s}$ in (11) we obtain

$$\lim_{t \to \infty} \frac{Y}{P} = \frac{r}{\alpha s} \qquad (12)$$

Since in the present case $\sigma = s$

$$\lim_{t \to \infty} \frac{Y}{P} = \frac{r}{\alpha \sigma} \qquad (13)$$

The expression

$$\theta = \frac{r}{\alpha\sigma} \qquad \text{——— (14)}$$

θ is called the coefficient of utilization.

When the economy grows at the equilibrium rate, so that $r = \alpha\sigma$, productive capacity is fully utilized. But as r falls below $\alpha\sigma$, a fraction of capacity $(1-\theta)$ is left unused.

Case II $\quad \sigma < s$.

[Note $\quad \sigma < s \Rightarrow \dfrac{dP/dt}{dK/dt} < \dfrac{P}{K}$

$$\Rightarrow \frac{1}{P}\frac{dP}{dt} < \frac{1}{K}\frac{dK}{dt}$$

$\Rightarrow$ rate of growth of productive capacity < rate of growth of capital stock

$\Rightarrow$ unutilized capital stock]

$$I\sigma = \frac{dP}{dt} \quad \text{and} \quad \frac{P}{K} = s$$

$$\Rightarrow P = sK$$

$$\Rightarrow \frac{dP}{dt} = s\frac{dK}{dt} = sI$$

$sI \Rightarrow$ If investment is I new project with productive capacity of Is are built

Since the productive capacity of the whole economy increases by $I\sigma$.

Thus productive capacity is reduced by $I(s-\sigma)$.

Thus, every year an amount of capital equal to $\frac{I(s-\sigma)}{s}$ becomes useless

[Note: $I(s-\sigma) = sI - \sigma I = \frac{dP}{dt}$ (net)

$$\therefore \frac{I(s-\sigma)}{s} = \frac{dP/dt}{\frac{dP/dt}{dK/dt}} = \frac{dK}{dt} \;]$$

The annual addition to capital = [investment minus capital losses] is

$$\frac{dK}{dt} = I - \frac{I(s-\sigma)}{s} = I\frac{\sigma}{s} \qquad \text{——— (15)}$$

and

$$K = K_0 + I_0\frac{\sigma}{s}\int_0^t e^{rt'}dt'$$

$$= K_0 + I_0\frac{\sigma}{sr}(e^{rt}-1) \qquad \text{——— (16)}$$

As t becomes large

$$K = I_0\frac{\sigma}{sr}e^{rt}$$

we know, $\frac{dY}{dt} = \frac{1}{\alpha}\frac{dI}{dt}$

$$\Rightarrow dY = \frac{1}{\alpha}dI$$

$$\Rightarrow Y = \frac{1}{\alpha}.I \Rightarrow Y = \frac{1}{\alpha}I_0e^{rt}$$

Thus

$$\frac{Y}{K} = \frac{\frac{1}{\alpha}I_0e^{rt}}{K_0 + I_0\frac{\sigma}{sr}(e^{rt}-1)}$$

$$\Rightarrow \lim_{t\to\infty}\frac{Y}{K} = \frac{r}{\alpha}.\frac{s}{\sigma} \qquad \text{——— (17)}$$

Using the fact $K = \frac{P}{s}$ we find from (17)

$$\lim_{t\to\infty}\frac{Y}{P} = \frac{r}{\alpha\sigma} \qquad \text{——— (18)}$$

which is exactly the same result as that of (13).

In Domar's model we also have a situation of 'knife edge'

From (14) we have

$$\theta = \frac{r}{\alpha\sigma}$$

$\theta \gtreqless 1$ according as $r \gtreqless \alpha\sigma$.

We can refer to r as the ~~'required rate'~~ 'actual rate' of growth of investment and $\alpha\sigma$ as the 'required rate' of growth of investment.

When $\theta > 1 \Rightarrow$ actual rate $>$ required rate

[Note required rate $\Rightarrow$ equilibrium rate]

$\Rightarrow$ a shortage of capacity

When $\theta < 1 \Rightarrow$ actual rate $<$ required rate
$\Rightarrow$ a surplus of capacity.

Equation (3) $\Rightarrow \frac{dY}{dt} = \frac{1}{\alpha}\frac{dI}{dt}$

$$= \frac{r}{\alpha} . I_0 e^{rt}$$

Equation (2) $\Rightarrow \frac{dP}{dt} = \sigma I = \sigma I_0 e^{rt}$

$$\frac{dY/dt}{dP/dt} = \frac{r}{\alpha\sigma} = \theta$$

When actual rate $>$ required rate $\Rightarrow r > \alpha\sigma$

$\Rightarrow \frac{dY}{dt} > \frac{dP}{dt}$

$\Rightarrow$ demand side effect $>$ capacity effect (supply side effect)

$\Rightarrow$ shortage of capacity.

Conversely if $r < \alpha\sigma$

$\Rightarrow$ deficiency of aggregate demand

$\Rightarrow$ surplus of capacity

The <u>paradoxical</u> result is that if investment actually grows at a 'faster' rate than required ($r > \alpha\sigma$) the end result would be a <u>shortage</u> rather than a surplus of capacity. It is also <u>paradoxical</u> if the actual growth of investment lags behind the required rate ($r < \alpha\sigma$) as the end result would be a <u>surplus</u> of capacity rather than shortage.

The logic behind this, as explained by Domar, is simple. When $r > \alpha\sigma$ the emergent capacity shortage will motivate an even faster rate of investment. But this would mean an increase in r, instead of the reduction called for under the circumstances. Consequently, the discrepancy between the two rates of growth would be intensified rather than reduced. This is the 'knife edge' problem in Domar's model. Any deviation from such a 'knife edge' time path will bring about a persistent failure to satisfy the norm of full utilization.