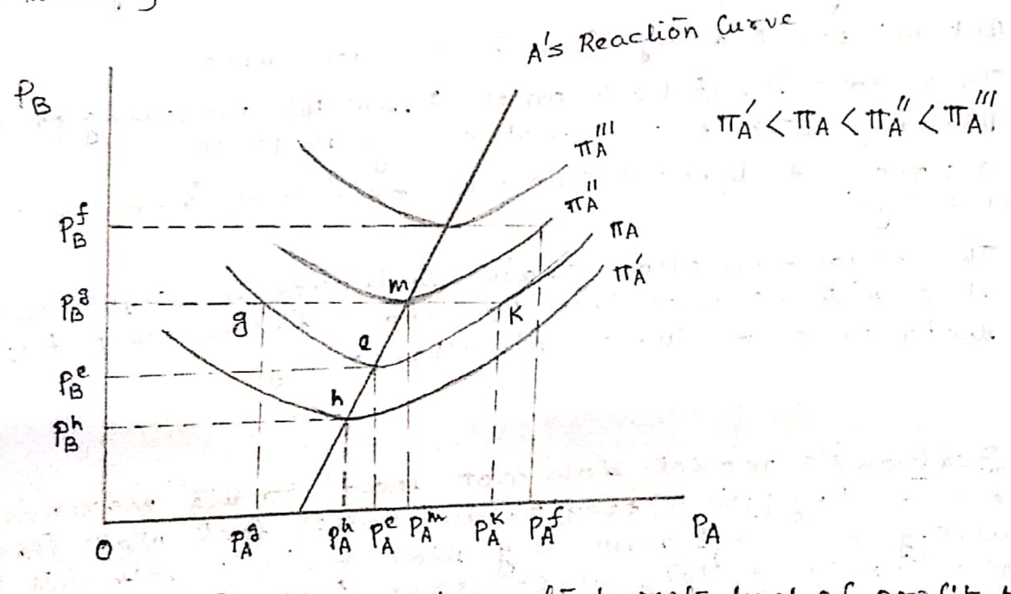
BERTRAND'S DUOPOLY MODEL

J. Bertrand developed his duopoly model in 1883. His model differs from Cournot's in that he assumes that each firm expects that the rival will keep its price constant irrespective of its own decision about pricing. Thus each firm is faced by the same market demand, and aims at the maximisation of its own profit on the assumption that the price of the competitor will remain constant.

The model is presented with the analytical tools of the reaction curves derived from the isoprofit maps. An isoprofit curve of a firm, say A, represents same level of profits which will accrue to firm A from various levels of prices charged by it and its rival firm. The isoprofit curve of firm A is convex to the axis that measures the prices charged by it (P_A). Such shape of this curve indicates that the firm must lower the price charged by it to a certain level to meet the price cutting by its rival firm in order to maintain its profits at a certain level, say π_A. If rival firm continues to cut price, firm A will not be able to maintain its level of profit at π_A any longer and would have to climb down to a lower isoprofit curve π_A'. In the same way, isoprofit curves for B too can be explained. The behaviour of isoprofit curves in Bertrand's model is exactly opposite of the behaviour of the isoprofit curves in Stackelberg's analysis of the Cournot's model. Here, the firms resort to marginal cost pricing and attempt equilibrium through maximisation of individual profit rather than joint profit.



The lowest isoprofit curve represents lowest level of profit to A and the highest; the highest level of it to it.

A's profit $\pi_A \Rightarrow g(P_A^g, P_B^g), e(P_A^e, P_B^e), K(P_A^K, P_B^g)$ and $f(P_A^f, P_B^f)$

B sets at P_B^g and A sets either at P_A^g or P_A^K

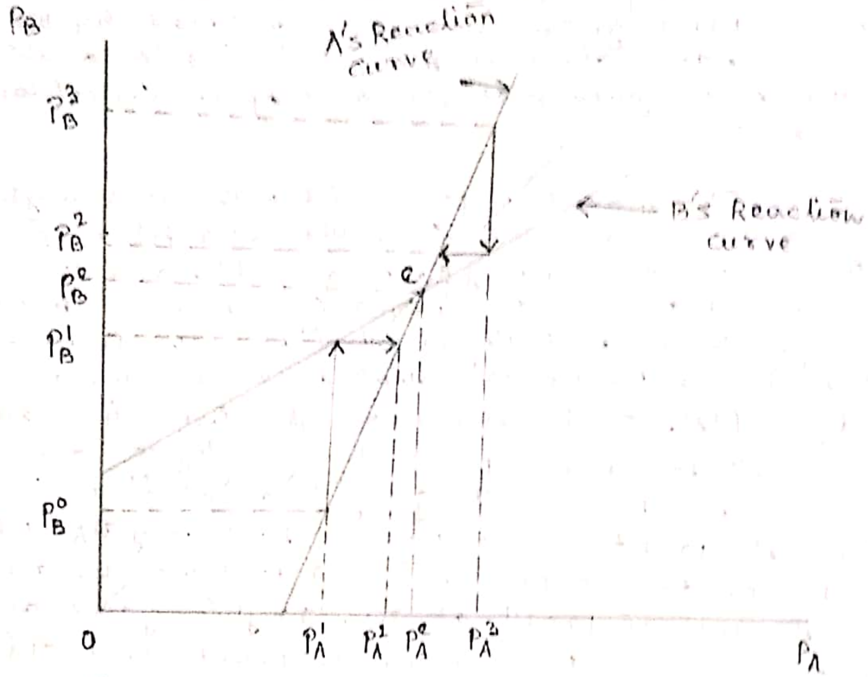
Here, $P_A^g < P_A^K \Rightarrow$ Sales at $P_A^g >$ Sales at $P_A^K \Rightarrow$ same level of Profit π_A

A sets price at P_A^m ($P_A^m < P_A^K$) and B sets at $P_B^g \Rightarrow$ Higher isoprofit curve or higher profit $\Rightarrow \pi_A''$ (as sale at $P_A^m >$ Sale at P_A^K)

If B reduces price to $P_B^e \Rightarrow$ Sale ↑es

A also reduces price to P_A^e from $P_A^K \Rightarrow$ Sale ↑es (otherwise sale decreases)

The locus of the points of the lowest prices set by B on each of A's isoprofit curves (points h, e, m etc) gives A's reaction curve, which is an upward sloping straight line.



The reaction curves of the two duopolists, as shown in the above figure, intersect each other at e, the equilibrium point. Any deviation from it sets in motion the process of adjustment that restore it to e.

Initially (P_A^1, P_B^0) — B is not on its reaction curve while A is.

To get back to it, B sets a Price at P_B^1, thinking that A will not change its price.

But, this puts A away from its reaction curve.

To restore itself back on it, A sets the matching price P_A^2, thinking that B will not change its price.

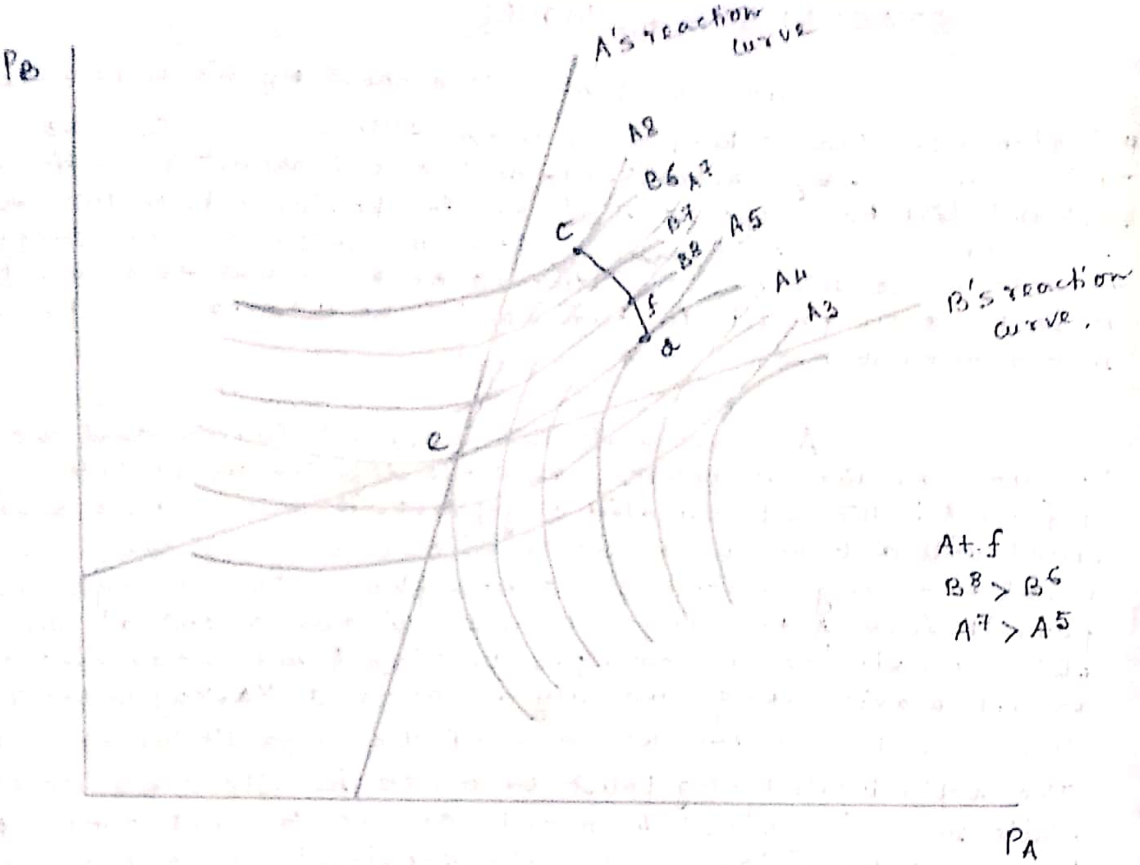
A's move disturbs B who feels detached from its reaction curve again.

The action-reaction chain continues until the two land at e with no incentive to depart from since two regain their reaction curves simultaneously.

Bertrand's model does not lead to the maximisation of the industry (joint) profit, due to the fact that firms behave naively, by always assuming that their rivals will keep its price fixed and they never learn from past experience which showed that the rival did not infact keep its price constant. The industry profit could be increased if firms recognised their past mistakes and abandoned the Bertrand pattern of behaviour.

If firms moved on any point between c and d on the Edgeworth contract curve one or both firms would have higher profits and hence industry profits would be higher.

At c ⇒ B same profit and A higher profit
At d ⇒ B higher profit and A same profit
Between c and d say at f ⇒ Both would have higher profit.



Criticisms

(1) Firms never learn from past experience
(2) Each firm maximises its own profit, not joint or industry's profit
(3) The model is closed - does not allow entry.

THE BERTRAND MODEL AND NASH EQUILIBRIUM

In 1883, i.e. 45 years after the publication of Cournot's book, Joseph Bertrand argued that in oligopolistic markets firms act as price setters rather than as quantity setters. In the Bertrand model each firm believes that it can charge whatever prices it desires, while the other firms continue to charge the same price as before. whereas Cournot firms believe that the rivals will keep their quantities constant, Bertrand firms believe that they will keep their prices constant. Now, prices instead of output are the instruments controlled by firms.

Here we will discuss Bertrand Model under homogeneous products and also under heterogeneous products. We first consider Bertrand Model with homogeneous products.

Bertrand Model with Homogeneous Products and Nash Equilibrium :-

To illustrate the Bertrand model let us consider a two firm homogeneous good industry (so that the industry is duopoly) where each firm produces the good at a constant per unit cost of c; i.e. the cost function of ith firm is $C_i(q_i) = cq_i$, $i = 1, 2$. There is no fixed cost. Then $AC = MC = c$. The industry demand function is $q = D(P)$. The firms are denoted by subscripts 1 and 2.

Given that consumers are perfectly informed of offered prices, they all will buy from firm 1 if $P_1 < P_2$. But if $P_1 > P_2$ then firm 1 sells nothing and all consumers buy from firm 2. Consumers are indifferent between the two firms if $P_1 = P_2$. In this case we can assume without loss of generality that firm 1 gets half of the consumers. Let $D(P)$ be the market demand function. Then firm 1's output demand function is

$$
\begin{aligned}
d_1(P_1, P_2) &= D(P_1) \quad \text{if } P_1 < P_2 \\
&= \tfrac{1}{2} D(P_1) \quad \text{if } P_1 = P_2 \\
&= 0 \quad \text{if } P_1 > P_2
\end{aligned}
$$

The profit function of firm 1 is

$$
\begin{aligned}
\pi_1(P_1, P_2) &= D(P_1)(P_1 - c) \quad \text{if } P_1 < P_2 \\
&= \tfrac{1}{2} D(P_1)(P_1 - c) \quad \text{if } P_1 = P_2 \\
&= 0 \quad \text{if } P_1 > P_2
\end{aligned}
$$

We can derive firm 2's output demand function and profit function analogously.

We now investigate the properties of the Nash equilibrium in prices. A Nash equilibrium is a pair of prices (P_i^*, P_j^*) such that

$$\pi_i(P_i^*, P_j^*) \geq \pi_i(P_i, P_j^*) \qquad i \neq j, \quad i, j = 1, 2$$

The Bertrand Paradox states that in equilibrium, firms will charge equal, competitive prices, and therefore make zero profits.

$$P_1^* = P_2^* = c$$

It is quite easy to understand the proof of this result. The proof consists in showing that neither unequal prices nor prices that are equal, but not equal to c can form a Nash equilibrium. In either case, at least one of the firms will have an incentive to change its price, more specifically, to undercut the other. Moreover, if the price are equal to each other and to c, such a pair will form a Nash equilibrium.

The intuition behind this result is simple. When both firms charge c, neither firm earns profits by deviating. If a firm lowers its price, it earns negative profit (as $P < c = MC = AC$). If either firm raises its price, it makes no sales at all. Thus $P_1^* = P_2^* = c$ is the only Bertrand (Nash) equilibrium. In this equilibrium, both firms charge a price equal to marginal cost (and also equal to average cost) and earn zero profit.

It is thus same as the social optimum (Competitive equilibrium). In the Bertrand equilibrium, with homogeneous product, no firm enjoys market power.

Bertrand Model with Heterogenous Products and Nash Equilibrium :-

We Consider the case of differentiated products. If firm 1 and 2 choose prices P_1 and P_2 respectively, the quantity that consumers demand from ith firm is

$$q_i(P_i, P_j) = a - P_i + b P_j \qquad i \neq j \quad i,j = 1,2$$

Here $b > 0 \Rightarrow$ the extent to which firm i's product is a substitute of firm j's product.

We assume that the total cost to firm i of producing q_i is

$$C_i(q_i) = c\, q_i \qquad i = 1,2$$

There is no fixed cost of Production and the marginal cost is constant at c, where $c < a$, and that the firms act simultaneously.

To find out the Nash equilibrium we define the strategy space as $S_i = [0, \infty]$

The profit to firm i when it chooses the price P_i and its rival chooses the price P_j is

$$\pi_i(P_i, P_j) = q_i(P_i, P_j)\,[P_i - c]$$
$$= (a - P_i + b P_j)(P_i - c)$$

Thus in the Bertrand model, the price pair (P_i^*, P_j^*) is a Nash equilibrium, if for each firm i, P_i^* solves

$$\underset{0 \leq P_i < \infty}{\text{Max}}\ \pi_i(P_i, P_j^*) = \underset{0 \leq P_i < \infty}{\text{Max}}\ (a - P_i + b P_j^*)(P_i - c)$$
$$= \underset{0 \leq P_i < \infty}{\text{Max}}\ [a P_i - P_i^2 + b P_i P_j^* - ac + c P_i - c b P_j^*)$$

The first order Condition for ith firm's optimisation

$$\frac{\partial \pi_i}{\partial P_i} = a - 2P_i + bP_j^* + bP_i \frac{\partial P_j^*}{\partial P_i} + c - cb \frac{\partial P_j^*}{\partial P_i} = 0$$

$$\Rightarrow a - 2P_i + bP_j^* + c = 0 \qquad \left[\because \frac{\partial P_j^*}{\partial P_i} = 0 \text{ i.e. zero Conjectural variation}\right]$$

[illegible], $j = 2$

$$P_i^* = \frac{1}{2}(a + bP_j^* + c)$$

Therefore if the price pair (P_1^*, P_2^*) is to be a Nash equilibrium, the firm's price choices must satisfy

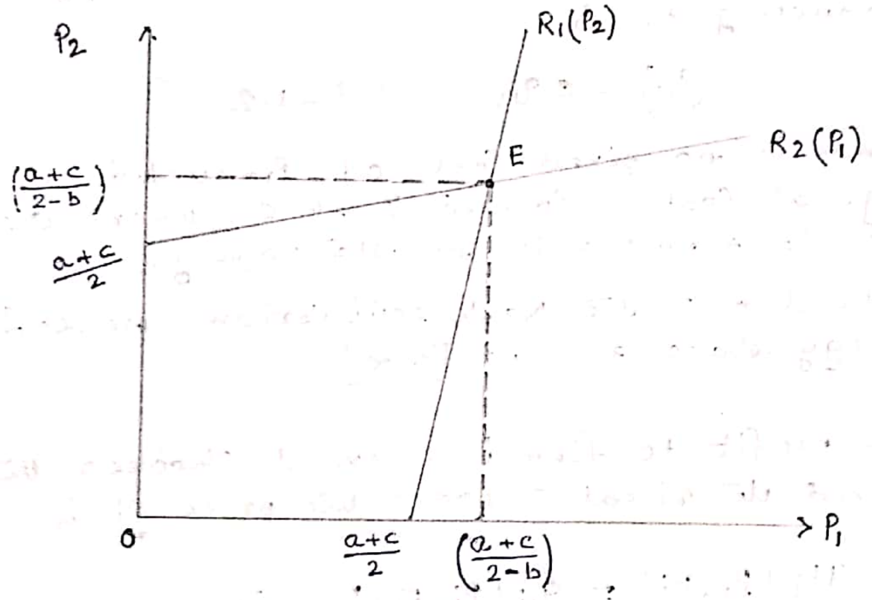
$$\left.\begin{aligned} P_1^* &= \tfrac{1}{2}(a + bP_2^* + c) \text{ and} \\ P_2^* &= \tfrac{1}{2}(a + bP_1^* + c) \end{aligned}\right\} \text{Reaction functions in prices.}$$

Solving this pair of equation yields

$$P_1^* = P_2^* = \frac{a+c}{2-b}$$

For $0 < b < 2$ and $c < a$

we get $\pi_i^*(P_i^*, P_j^*) > 0$ as $P_i^* > c$

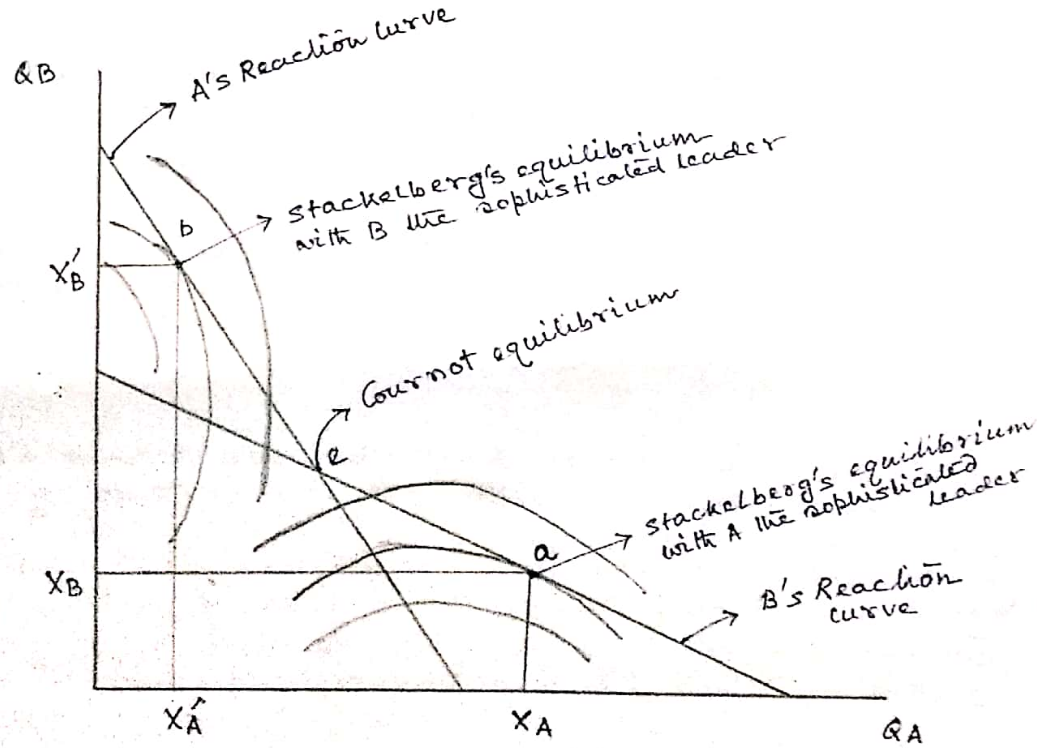


It is clear that increasing product heterogeneity increases the level of profit for both firms under price competition (and quantity competition - Cournot model). This explains why firms try to differentiate their product from each other

STACKELBERG MODEL

This model was developed by the German economist Heinrich von Stackelberg and is an extension of Cournot's model. It is assumed, by Stackelberg, that one duopolist is sufficiently sophisticated to recognise that his competitor acts on the Cournot assumption. This recognition allows the sophisticated duopolist to determine the reaction curves of his rival and incorporate it in his own profit function, which he then proceeds to maximise like a monopolist.

Assume that the isoprofit curves and the reaction functions of the duopolists are those depicted in the following figure. If firm A is the sophisticated oligopolist, it will assume that its rival will act on the basis of its own reaction curve, i.e. it's rival is acting on Cournot's assumption. This recognition will permit firm A to choose to set its own output at the level which maximises its own profit. Firm A thus determine its profit maximising quantity at point $a(X_A, X_B)$ where reaction curve of Firm B touches one of the isoprofit curves of Firm A. The sophisticated oligopolist becomes in effect the leader, while the naive rival who acts on the Cournot assumption becomes the follower. Clearly sophistication is rewarding for A because he reaches an isoprofit curve closer to his axis than if he behaved with the same naivete as his rival. The naive follower is worse off as compared with the Cournot equilibrium, since with this level of output he reaches an isoprofit curve further away from his axis. Opposite is the case (X_A', X_B') when Firm B acts as a sophisticated leader.



If both the firms behave as sophisticated quantity leader, each will attempt maximisation of its own profit like a monopolist. The market situation will then unstable leading to what is called Stackelberg's disequilibrium. This will initiate either a price war between the two continuing until one of them surrenders and turns a follower or a situation arises in which the two enter into collusion.

Stackelberg's model has one important implication, one of which is that the follower ultimately realises that its naive behaviour fails to pay-off. In consequence, a high degree of interdependence between them emerges.

This model shows that naive behaviour does not pay. The rivals should recognise their interdependence. By recognising the reactions of the rival each duopolist can reach a higher level of profit for himself. If both firms start recognising their mutual interdependence, each starts worrying about the rivals profit and rival's reaction. If each ignores the other, a price war will be inevitable, as a result of which both will be worse off. The model also shows that a bargaining procedure and a collusive agreement becomes advantageous for both the duopolists. With such a collusive agreement the duopolists may reach a point on the contract curve where the profit levels are higher for both of them.