Main Steps for derivation of Second Isomorphism theorem

Consider the mapping $\phi : H \longrightarrow HN/N$ given by

$$\phi(h) = hN \in HN/N.$$

Then prove that 1) ϕ is a homomorphism

2) ϕ is onto

Next by the virtue of Fundamental theorem on homomorphism,

$$H/\ker\phi \simeq HN/N$$

Thereafter, Show that $\ker\phi = H \cap N$.

Consequently, we reach the final conclusion :

$$H/H\cap N \simeq HN/N$$

Third Isomorphism theorem

Let $(G,*)$ be a group and $H \subset N \subset G$ such that

$(H,*)$ is normal subgroup in $(N,*)$

$(N,*)$ is normal subgroup in $(G,*)$

$(H,*)$ is normal subgroup in $(G,*)$

Then i) N/H is normal in G/H

and ii) $(G/H)/(N/H) \simeq G/N$.