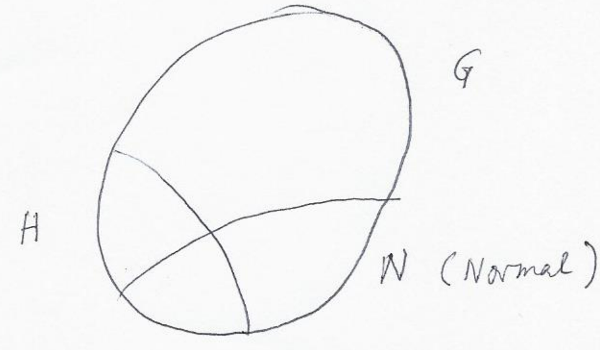

$\boxed{N \triangleleft G}$

$$HN = \{h*n : h \in H, n \in N\} \subset G$$

$$H = \{h*e : h \in H\} \subset HN \quad \text{since } e \in N$$

$$N = \{e*n : n \in N\} \subset HN \quad \text{since } e \in H$$

A trivial inclusion ; $N \subset HN \subset G$

- $\boxed{N \triangleleft HN}$

Trivial.

$N \triangleleft G$

$gN = Ng \quad \forall g \in G$

$\therefore gN = Ng \quad \forall g \in HN$

$gNg^{-1} = N$

$\boxed{HN \text{ is a Subgroup of } G}$

$g_1 = h_1 n_1 \qquad g_2 = h_2 n_2$

$g_1 g_2 = h_1 n_1 h_2 n_2$

$= h_1 h_0 n_0 n_2$

$= h_3 n_3 \in HN$

$(hn)^{-1} = n^{-1} h^{-1} \in NH = HN$

$\therefore HN$ is a Subgroup of G.

$HN = NH$

$hnh^{-1} \in N$

~~$\Rightarrow hn \in Nh$~~

$hnh^{-1} = n_0$

$hn = n_0 h \in NH$

$HN \subset NH$

$N \triangleleft G$

$gng^{-1} \in N$

$\forall g \in G$

$\forall n \in N$

$h^{-1} n h \in N$

$h^{-1} n h = n_1$

$nh = hn_1 \in HN$

$NH \subset HN$

$HN \subset NH \subset HN$

$\Rightarrow HN = NH$

- $H \cap N$ is normal in H, $\boxed{H \cap N \triangleleft H}$

$M = H \cap N \subset H, N$

$hmh^{-1} \in H$ (Trivial)

$hmh^{-1} \in N$

$\therefore hmh^{-1} \in H \cap N$