STACKELBERG MODEL OF DUOPOLY: A DYNAMIC GAME WITH COMPLETE INFORMATION

In the Stackelberg model, one firm (the 'leader' or 'dominant') gets to choose the quantity first and then the other firm (the 'follower') selects the level of its output. The leader then enjoys a first-mover advantage in that it can anticipate the actions of the follower and make its optimal choice accordingly. Here the movements are sequential whereas in Cournot the movements are simultaneous. So there are different stages of the game which we refer to as dynamic game and we solve it by the method of backward induction.

In Stackelberg model a dominant or leader firm moves first and a subordinate or follower firm moves second. Following Stackelberg we will develop the model under the assumptions that the firms choose quantities, as in Cournot model.

The timing of the game is as follows.

(1) Firm 1 chooses a quantity $q_1 \geq 0$

(2) Firm 2 observes q_1 and then chooses a quantity $q_2 \geq 0$.

(3) The pay-off to firm i is given by the profit function

$$\pi_i(q_i, q_j) = q_i\,[P(Q) - c] \qquad \begin{matrix} i \neq j \\ i,j = 1,2 \end{matrix}$$

where $P(Q) = a - Q \rightarrow$ the linear demand function

$$Q = q_1 + q_2$$

The total cost to firm i of producing quantity q_i is $C_i(q_i) = c\,q_i \Rightarrow$ there is no fixed cost

and the marginal cost is constant at c

To solve for the backward-induction outcome of this game, we first compute firm 2's reaction to an arbitrary quantity by firm 1.

$$\max_{q_2 \geq 0} \pi_2(q_1, q_2) = \max_{q_2 \geq 0} q_2\,[P(Q) - c]$$

$$= \max_{q_2 \geq 0} q_2\,[a - q_1 - q_2 - c]$$

$$\pi_2 = aq_2 - q_1q_2 - q_2^2 - cq_2$$

$$\frac{d\pi_2}{dq_2} = a - q_1 - 2q_2 - c = 0 \qquad \left[\text{since } \frac{dq_1}{dq_2} = 0\right]$$

$$\Rightarrow a - q_1 - c = 2(q_2)$$

$$\therefore q_2 = \frac{a - q_1 - c}{2}$$

Thus the reaction function of firm 2 is

$$R_2(q_1) = \frac{a - q_1 - c}{2} \quad \text{provided } q_1 < a - c$$

Firm 1 realises that for any q_1 it selects, firm 2's output will be chosen from the reaction function and the profit for the associated quantities will be realised.

Since firm 1 can solve firm 2's problem as well as firm 2 can solve it, firm 1 should anticipate that the quantity choice q_1 will be met with the reaction function $R_2(q_1)$.

Therefore, firm 1 maximises its profit given the constraint $R_2(q_1) = \frac{1}{2}(a - q_1 - c)$. Given this constraint, firm 1's problem is to maximise its profit, i.e.

$$\max_{q_1 \geq 0} \pi_1(q_1, R_2(q_1)) = \max_{q_1 \geq 0} q_1\left[a - q_1 - R_2(q_1) - c\right]$$

$$= \max_{q_1 \geq 0} q_1\left[a - q_1 - c - \frac{1}{2}(a - q_1 - c)\right]$$

$$= \max_{q_1 \geq 0} q_1 \cdot \frac{a - q_1 - c}{2}$$

$$\therefore \pi_1 = \frac{1}{2}(aq_1 - q_1^2 - cq_1)$$

The first order condition $\frac{d\pi_1}{dq_1} = \frac{1}{2}(a - 2q_1 - c) = 0$

$$\therefore q_1^* = \frac{a - c}{2}$$

$$\therefore R_2(q_1^*) = \frac{a - c}{2} - \frac{1}{2} \cdot \left(\frac{a - c}{2}\right)$$

$$= \frac{a - c}{4} = q_2^*$$

This is the backward induction outcome of the Stackelberg duopoly game.

In Case of Nash equilibrium of the Cournot model each firm produces $\frac{a-c}{3}$. The aggregate quantity is $\frac{2(a-c)}{3}$.

Thus $Q^{Cournot} = \frac{2(a-c)}{3} = 0.66\,(a-c)$

The aggregate quantity in the backward induction outcome of the stackelberg game is

$$
\begin{aligned}
Q^{Stackelberg} &= q_1^* + q_2^* \\
&= \frac{a-c}{2} + \frac{a-c}{4} \\
&= \frac{3(a-c)}{4} = 0.75\,(a-c)
\end{aligned}
$$

$\therefore \boxed{Q^{Stackelberg} > Q^{Cournot}}$

<u>The Cournot Case</u>

$$
\begin{aligned}
P &= a - Q \\
&= a - \frac{2(a-c)}{3} \\
&= \frac{3a - 2a + 2c}{3} = \frac{a+2c}{3}
\end{aligned}
$$

Thus, $P^{Cournot} = \frac{a+2c}{3}$

<u>The Stackelberg Case</u>

$$
\begin{aligned}
P &= a - Q \\
&= a - \frac{3(a-c)}{4} \\
&= \frac{4a - 3a + 3c}{4} = \frac{a+3c}{4}
\end{aligned}
$$

Thus, $P^{Stackelbarg} = \frac{a+3c}{4}$

Now, $P^{Stackelberg} - P^{Cournot}$

$$
\begin{aligned}
&= \frac{a+3c}{4} - \frac{a+2c}{3} \\
&= \frac{3a+9c-4a-8c}{12} \\
&= \frac{-a+c}{12} \\
&= -\frac{(a-c)}{12} < 0 \quad [\because c < a]
\end{aligned}
$$

$\therefore \boxed{P^{Stackelberg} < P^{Cournot}}$

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Profit in Cournot model

$$\pi_1 = q_1(P - c)$$

$$= \frac{a-c}{3}\left(\frac{a+2c}{3} - c\right) \quad \left[\text{as } p^{Cournot} = \frac{a+2c}{3}\right]$$

$$= \frac{(a-c)}{3} \cdot \frac{(a+2c-3c)}{3}$$

$$= \frac{(a-c)}{3} \cdot \frac{(a-c)}{3} = \frac{(a-c)^2}{9}$$

And $\pi_2 = q_2(P - c)$

$$= \frac{(a-c)^2}{9} \quad \left[\text{As } q_1 = q_2 \text{ and } P(Q) = \frac{a+2c}{3}\right]$$

$$\therefore \pi = \pi_1 + \pi_2 = \frac{2}{9}(a-c)^2$$

$$= 0{\cdot}222\,(a-c)^2$$

Thus, $\pi^{Cournot} = 0{\cdot}222\,(a-c)^2$

Profit in Stackelberg model.

$$\pi_1 = q_1[P(Q) - c]$$

$$= \frac{(a-c)}{2}\left[\frac{a+3c}{4} - c\right] \quad \left[\text{as } P^*(Q) = \frac{a+3c}{4}\right]$$

$$= \frac{(a-c)}{2}\left[\frac{a+3c-4c}{4}\right]$$

$$= \frac{(a-c)}{2} \cdot \frac{(a-c)}{4} = \frac{(a-c)^2}{8}$$

$$\pi_2 = q_2[P(Q) - c]$$

$$= \frac{(a-c)}{4}\left[\frac{a+3c}{4} - c\right]$$

$$= \frac{(a-c)}{4} \cdot \left[\frac{a+3c-4c}{4}\right]$$

$$= \frac{(a-c)}{4} \cdot \frac{(a-c)}{4} = \frac{(a-c)^2}{16}$$

$$\therefore \pi = \pi_1 + \pi_2 = \frac{(a-c)^2}{8} + \frac{(a-c)^2}{16} = \frac{2(a-c)^2 - (a-c)^2}{16}$$

$$= \frac{3}{16}(a-c)^2 = 0{\cdot}1875\,(a-c)^2$$

$$\therefore \pi^{stackelberg} = 0{\cdot}1875\,(a-c)^2$$

Hence, $\boxed{\pi^{stackelberg} < \pi^{Cournot}}$

It is to be noted that

$$\pi_1^{Cournot} = \frac{(a-c)^2}{9}$$

and $$\pi_1^{stackelberg} = \frac{(a-c)^2}{8}$$

Thus, $$\boxed{\pi_1^{stackelberg} > \pi_1^{Cournot}}$$

Also,

$$\pi_2^{Cournot} = \frac{(a-c)^2}{9}$$

and $$\pi_2^{stackelberg} = \frac{(a-c)^2}{16}$$

Thus, $$\boxed{\pi_2^{stackelberg} < \pi_2^{Cournot}}$$

Observations

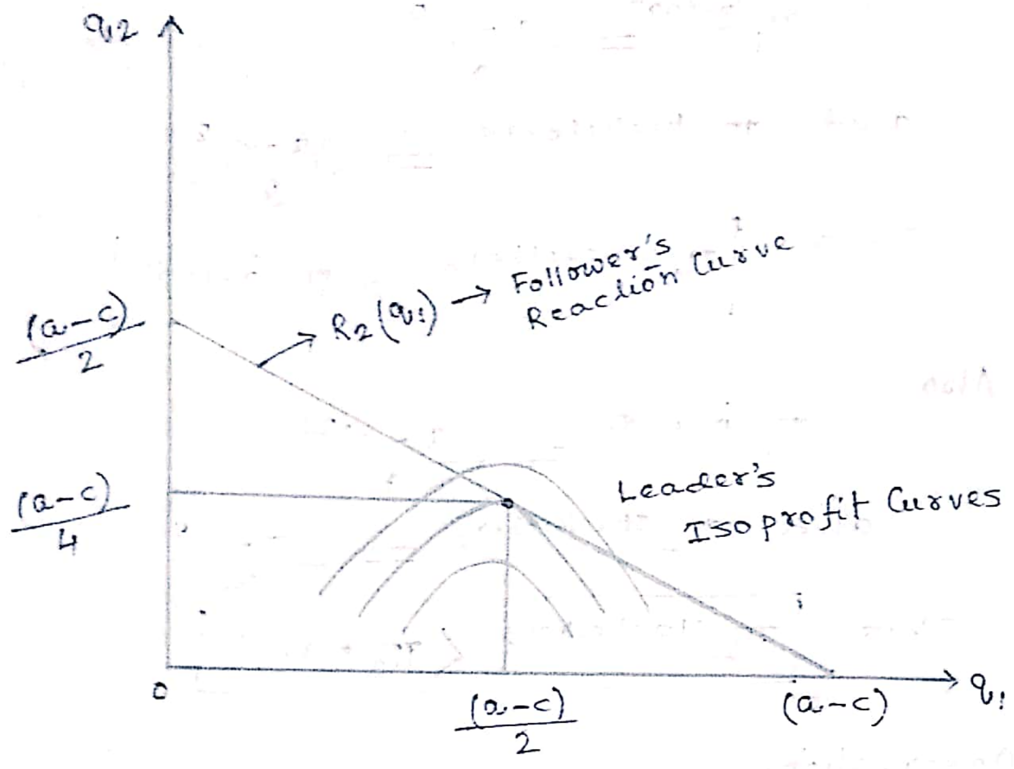
(1) In Stackelberg game, firm 1 could have achieved Cournot profit by choosing Cournot quantity. $\frac{(a-c)}{3}$

(2) In Stackelberg model firm 1 could have achieved its Cournot profit level but chose to do otherwise, so firm 1's profit in the stackelberg model must exceed its profit in the Cournot model.

(3) Market clearing price in the stackelberg model is lower than the Cournot market clearing price.

(4) Aggregate profit in stackelberg model is lower than aggregate profit in Cournot model (when firm 1 chooses Cournot quantity. similarly for firm 2).

Thus firm 1 is better off (and hence firm 2 is worse off) in the stackelberg model than in the Cournot model.



Diagrammatically, the Cournot-Nash solution is obtained at the intersection of the two reaction functions. But in the Stackelberg model, we have seen that the leader takes as given the follower's reaction function, and chooses to produce at the point where one of its iso-profit curves is tangential to the reaction function of the follower.