Linear Superposition Principle as a consequence (of double slit experiment)

Principle: *Any two (or more) quantum states can be added together (superposed) and the result will be another valid quantum state; and conversely, that every quantum state can be represented as a sum of two or more other distinct quantum states. Mathematically, it refers to a property of solutions to the Schrödinger equation; since the Schrödinger equation is linear, any linear combination of solutions will also be a solution.*

Explanation: The superposition principle is the idea that a system is in all possible states at the same time, until it is measured. After measurement it then falls to one of the basis states that form the superposition, thus destroying the original configuration. A classic example of this is the double-slit experiment. Here, two slits in a barrier allow for the passage of electrons. The result of this experiment is an interference pattern not predicted by classical mechanics.

Let us consider the double slit experiment with electrons again. The experimental arrangement consists of an electron gun which sends **identical** electrons through a screen which has two slits to a wall (screen). The electron gun produces the electrons **one by one**, so that at any given time there is only **one** electron traveling from the electron gun to the wall. We consider two different experiments with this arrangement, namely, one experiment in which a **measurement** is carried out to determine which slit the electron went through, and a second experiment in which **no measurement** is made to determine which slit the electron goes through. In both cases a large number, say 10^6, electrons are sent in, one by one, and the distribution of the positions at which the electrons hit the wall is measured.

a) Experiment with Detection:

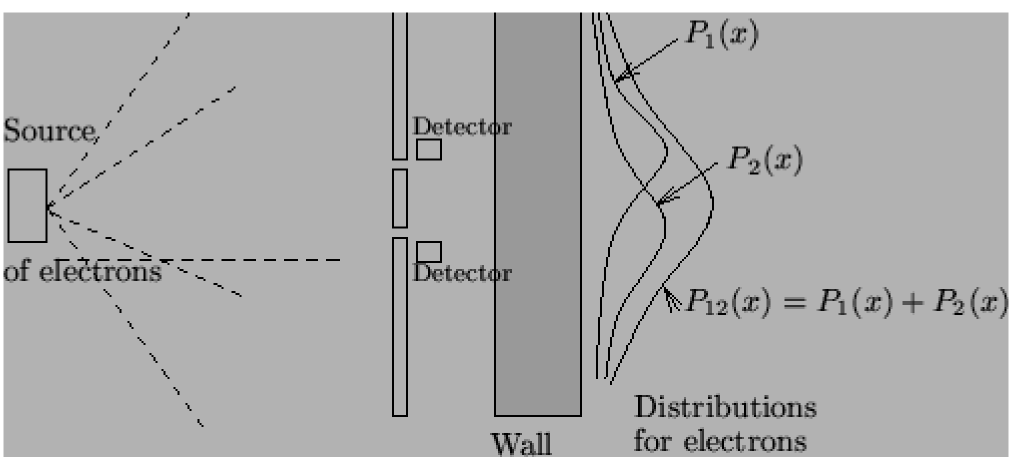

Figure 1: Electron with detectors

We perform the experiment as given in Figure 1 with **both** slits 1 and 2, open and with the **additional requirement** that we determine which slit the electron actually passes through. This can be arranged by fixing two detectors, say a light source, at the back of the slits as shown in Figure 1. Since we know which slit the electron goes through we can plot three distribution curves namely $P_1(x)$, $P_2(x)$ and $P_{12}(x)$. $P_1(x)$ and $P_2(x)$ are the distribution curves for electrons go through slit 1 and slit 2 respectively. Similar to the result obtained for bullets, the probability of the electron arriving at a point on the wall when both slits are open, denoted by $P_{12}(x)$ is given by

$$P_{12} = P_1 + P_2$$

We consequently have the result that when the electron's path is measured, it has a **particle-like** behavior.

b) Experiment without Detection:

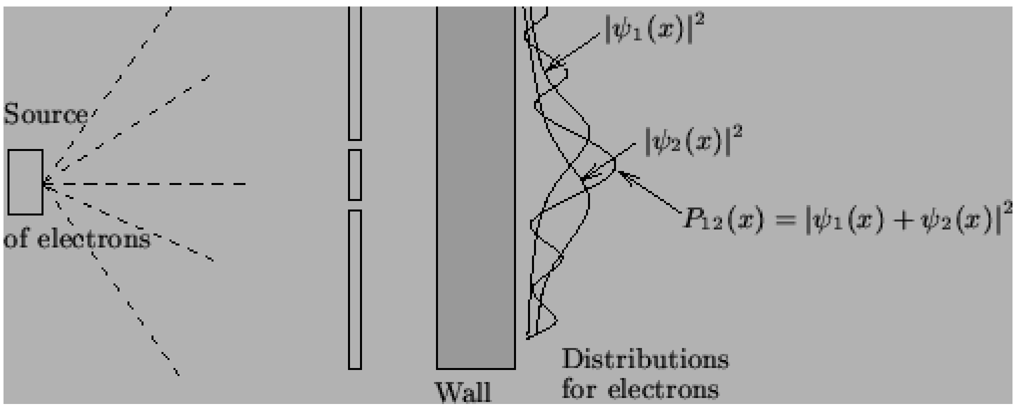

Figure 2: Electron without Detector

Consider now the same experiment as before, but with the detectors removed, as shown in Figure 2. In other words, we **do not** make any measurement to determine which slit the electron goes through. The result of this experiment is illustrated in Figure 2, and shows that even a **single** electron gives rise to an interference. The interference pattern P_{12} is exactly like I_{12} as obtained for light waves. This suggests that electrons behave like waves and we have to introduce a probability amplitude Ψ_1 for electrons when slit 2 is closed and an amplitude Ψ_2 for electrons when slit 1 is closed. We then have in analogy with waves

$$P_1 = |\Psi_1|^2$$

$$P_2 = |\Psi_2|^2$$

When **both** slits 1 and 2 are open, and no measurement is made, the resultant distribution P_{12} is the square of modulus of the sum of Ψ_1 and Ψ_2. The probability amplitudes obey the **superposition principle** when the different paths are **not known (without detector)**, and yield

$$\Psi_{12} = \Psi_1 + \Psi_2$$

It is the superposed amplitude Ψ_{12} that determines the outcome when no measurement is performed. Hence

$$P_{12} = |\Psi_{12}|^2 = |\Psi_1 + \Psi_2|^2$$
$$\neq |\Psi_1|^2 + |\Psi_2|^2 = P_1 + P_2$$

The superposition principle is the unique feature of quantum mechanics, and shows graphically that, under some circumstances, particles behave as interference waves. **Knowing** which slit the electron passes through, the electron exhibits particle-like behaviour; **not knowing** which slit the electron goes through, the electron exhibits wave-like behaviour. This is the famous wave-particle duality of quantum mechanics.

The superposition of states thus explains the quantum interference pattern. When both slits are open, the description of the system is the superposition of the states when each slot is opened individually and it is just this superposition that accounts for the interference. This is true until one tries to determine which path is taken by an electron, after which the state of the system collapses. The classical interpretation of particles bombarding a detector fails to adequately describe the situation.