Heisenberg Uncertainty Principle

Statement: *For a particle, it is impossible to determine both the position and its canonically conjugate momentum simultaneously and precisely.*

or

The product of uncertainties in determining the position and momentum of a particle at the same instant is at best of the order of ℏ/2

i.e.
$$\Delta x \Delta p_x \geq \frac{\hbar}{2}$$

where Δx *is the uncertainty in determining the position of the particle and* Δp_x *is that in determining the momentum.*

Example: ***The non-existence of the electron in the nucleus***

The radius of the nucleus of any atom is of the order of 10^{-14}m, so that if an electron confined within nucleus, the uncertainty in its position must be greater than 10^{-14}m. According to the uncertainty principle $\Delta x \Delta p_x \geq \hbar$, where $\hbar = 1.055 \times 10^{-34}$Js.

Now maximum uncertainty in position of electron within nucleus,

Δx = diameter of nucleus = 2×10^{-14}m.

So, $\Delta p_x \geq \dfrac{\hbar}{\Delta x} = 5.275 \times 10^{-21} kg - m/s$ is the uncertainty in momentum of the electron. The momentum of the electron must be at least-comparable with its magnitude, *i.e.*, $p_x = 2.637 \times 10^{-21} kg - m/s$.

The kinetic energy of the electron of mass m is given by $K.E. = \dfrac{p_x^2}{2m} = 97 MeV$.

This means that if the electrons exist inside the nucleus, their kinetic energy must be of the order of 97 MeV. But experimental observation shows that no electron in atom possess energy greater than 4 MeV. Clearly the inference is that the electrons *do not exist* in the nucleus.

Example

Consider an electron moving in an H-atom in 1S orbit with a speed of 2×10^{6} m/s.

$$p = 9.1\times10^{-31}\times2\times10^{6}\text{ m/s} = 18.22\times10^{-25}\text{ kg m/s}.$$

If this is measure with an accuracy of 1%, the uncertainty of mom. will be

$$\Delta p_x = 18.22\times10^{-25}\times0.01$$
$$= 18.22\times10^{-27}$$

∴ Uncertainty of position

$$\Delta x = h/\Delta p = \frac{6.626\times10^{-34}}{18.22\times10^{-27}} = 3.64\times10^{-8}\text{ m}$$
$$= 36\text{ nm}.$$

Radius of 1st orbit in H-atom $= 0.529\,\text{Å} = 0.0529$ nm

∴ The uncertainty of electron's position in H-atom will be

$$\frac{36}{0.0529} \simeq 700 \text{ times the radius of the atom itself}$$

— it is not even known whether at the given instance the electron is within the atom or not!

The path of the electron is not defined at all, it is spread out due to uncertainty — in contrary to the Bohr's concept.

Example

Ball of mass 0.2 kg., $v = 10$ m/s.

$$p = 2\text{ kg m/s}.$$

If the error in this measurement is 0.1%,

$$\Delta p = 2\times10^{-3}\text{ kg m s}^{-1}$$

∴ Uncertainty in position

$$\Delta x = h/\Delta p = 3.31\times10^{-31}\text{ m}.$$

↳ negligible.

∴ both p and x can be determined with sufficient accuracy.

HW: *Prove uncertainty relation for energy and time from position momentum uncertainty relation.*